HYGENE: A Diffusion-based Hypergraph Generation Method

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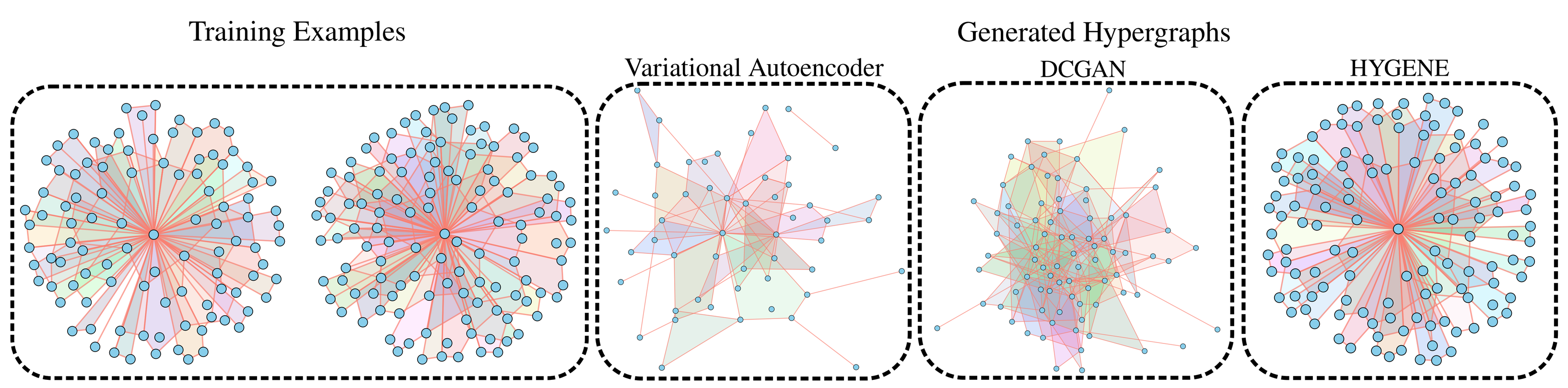
We introduce HYGENE, a hierarchical diffusion-based method for hypergraph generation.

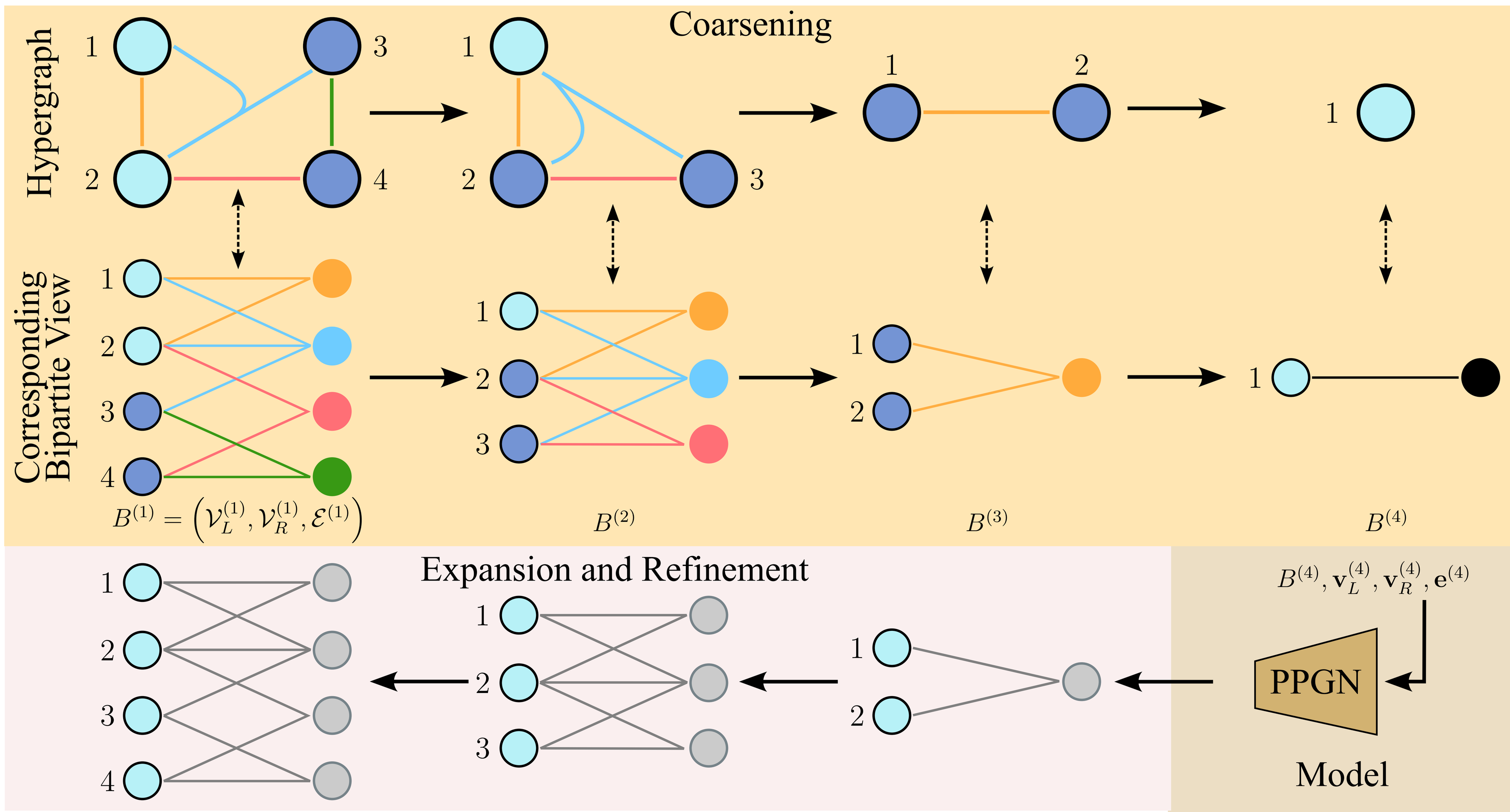
Motivation

- Hypergraphs (higher-order extension of graphs where *hyper*edges can contain more than two nodes) are more expressive than regular graphs.
- Broad field of application: 3D graphics (meshes), pharmaceutical research (molecules), electronics (circuit design).
- No method directly generates hypergraph and generating the incidence matrix as an image (using VAE, GAN and diffusion models) fails.
- Here we are interested in *unfeatured* hypergraphs. Our goal is to train a model able to sample a specific connectivity distribution.



Comparison of HYGENE with classical generation methods

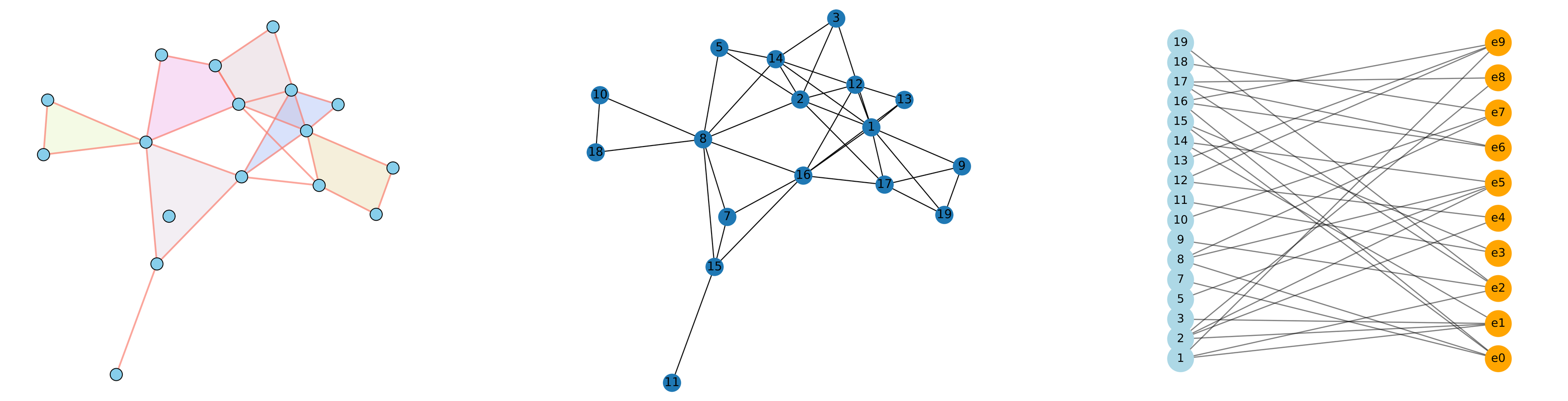
Pipeline



Like graphs [1], unzoom (lose details) and train a model to zoom (reconstruct details). Working on graph projections is easier than on hypergraphs.

Ideas

1. **Clique expansion:** Each hyperedge is collapsed into a clique. The clique expansion has the same spectral properties as its hypergraph, but recovering a hypergraph from its clique expansion is *NP-hard*. Used for coarsening.
2. **Star expansion:** The hypergraph is turned into a bipartite graph, where left side nodes correspond to the original nodes, and right side nodes to hyperedges. Each hyperedge is then connected to all of its nodes. Very easy to manipulate. Maintained in parallel during coarsening.



(a) Original hypergraph   (b) Clique expansion   (c) Star expansion

A hypergraph and its clique and star expansions

Generated examples

Train   Generated   Train   Generated   Train   Generated   Train   Generated   Train   Generated   Train   Generated

(a) Erdos-Renyi   (b) Stochastic Block Model   (c) Ego   (d) Tree   (e) ModelNet40 Plant meshes   (f) ModelNet40 Piano meshes

Numerical Results

Model	SBM Hypergraphs (n_{avg} = 31.73, std = 0.55)				Ego Hypergraphs (n_{avg} = 109.71, std = 10.23)				Tree Hypergraphs (n_{avg} = 32, std = 0)				Erdos-Renyi Hypergraphs (n_{avg} = 32, std = 0.07)			ModelNet40 Piano (n_{avg} = 177.29, std = 57.11)			ModelNet40 Plant (n_{avg} = 124.86, std = 87.88)		
	Valid SBM ↑	Node Deg ↓	Edge Size ↓	Spectral ↓	Valid Ego ↑	Node Deg ↓	Edge Size ↓	Spectral ↓	Valid Tree ↑	Node Deg ↓	Edge Size ↓	Spectral ↓	Node Deg ↓	Edge Size ↓	Spectral ↓	Node Deg ↓	Edge Size ↓	Spectral ↓	Node Deg ↓	Edge Size ↓	Spectral ↓
HyperPA	2.5%	4.062	0.407	0.273	0%	2.590	0.423	0.237	0%	0.315	0.284	0.159	5.530	0.183	0.177	9.254	**0.023**	**0.067**	6.566	0.046	0.061
VAE	0%	1.280	1.059	0.024	0%	0.803	1.458	0.133	0%	0.072	0.480	0.124	2.140	0.540	0.035	8.060	1.686	0.396	3.895	1.573	0.205
GAN	0%	2.106	1.203	0.059	0%	0.917	1.665	0.230	0%	0.151	0.469	0.089	2.560	0.657	0.048	409.0	86.38	0.697	378.1	56.35	0.364
Diffusion	0%	1.717	1.390	0.031	0%	3.984	2.985	0.190	0%	1.718	1.922	0.127	2.225	0.781	0.014	20.90	4.192	0.113	21.03	3.439	0.069
HYGENE	**65%**	**0.161**	**0.002**	**0.010**	**90%**	**0.063**	**0.220**	**0.004**	**77.5%**	**0.059**	**0.108**	**0.012**	**0.445**	**0.012**	**0.006**	**6.290**	0.027	0.117	**2.428**	**0.027**	**0.034**

Conclusions

- We introduced HYGENE, the first diffusion-based approach for hypergraph generation.
- Our work generalizes previous iterative local expansion schemes [1] and coarsening processes [2] to hypergraphs.
- By training a denoising diffusion model, we successfully validated the method's capability to generate hypergraphs from targeted distributions.

Next: Add features. Currently working on mesh generation.

[1] Bergmeister, A., Martinkus, K., Perraudin, N., Wattenhofer, R. (2023). *Efficient and scalable graph generation through iterative local expansion*

[2] Loukas, A. (2019). *Graph reduction with spectral and cut guarantees*

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